Probability Distribution Curves

and

Critical Values

$z_{\alpha/2}$ & $t_{\alpha/2}$

What is a **Probability Distribution**?

Probability Distribution is a description that gives the probability for each value, or a range of values of the random variable. The **Probability Distribution** for a random variable can be in the form of a table, graph, or formula.

What is a **Probability Distribution Curve**?

Probability Distribution Curve describes the shape of the distribution which depends on other factors related to the type of distribution.
The total area under such curves is always 1.

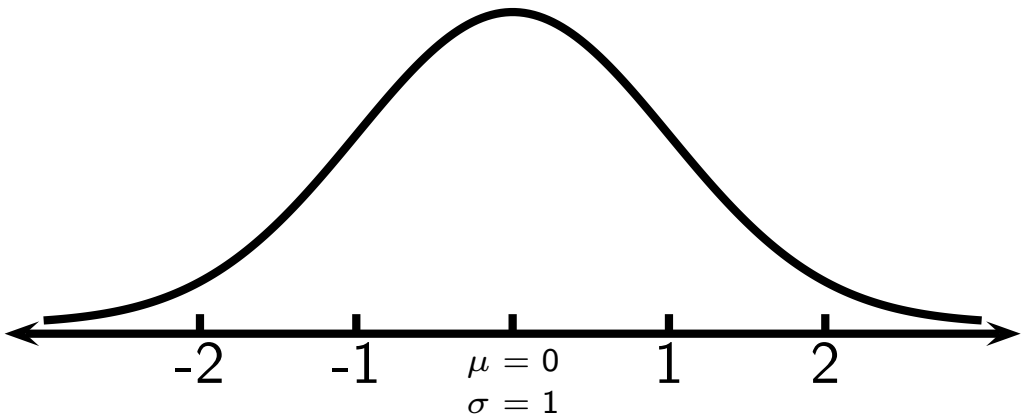
What are some common **Probability Distributions**?

- Standard Normal Distribution
- Student t-Distribution
- Chi-Square Distribution
- F-Distribution

Each one of these distributions will be used in different applications satisfying certain criteria that is required by the type of distribution.

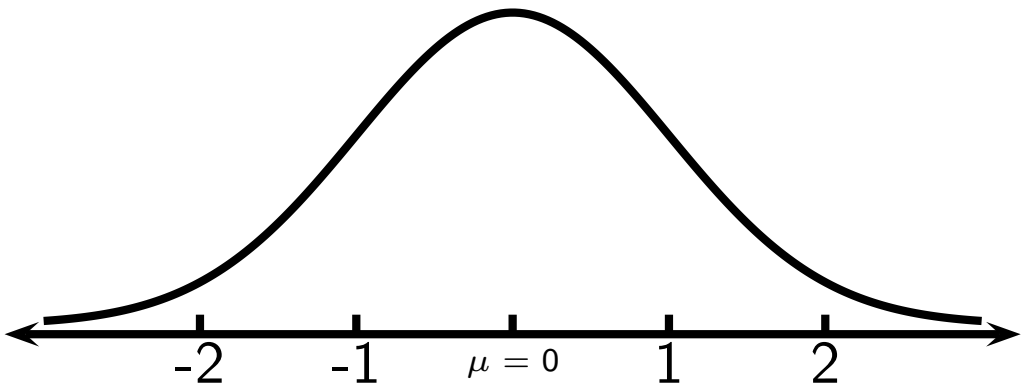
What is **Standard Normal Distribution**?

It is a normal probability distribution with $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1. The density curve is symmetric and bell-shaped.



What is **Student t-Distribution**?

It is a normal probability distribution with $\mu = 0$ and σ is unknown. The total area under its density curve is equal to 1. The density curve is symmetric and bell-shaped.



This distribution is often referred to as **t-Distribution**. The shape also depends on degrees of freedom for the collection of sample data.

What is a **Degrees of Freedom**?

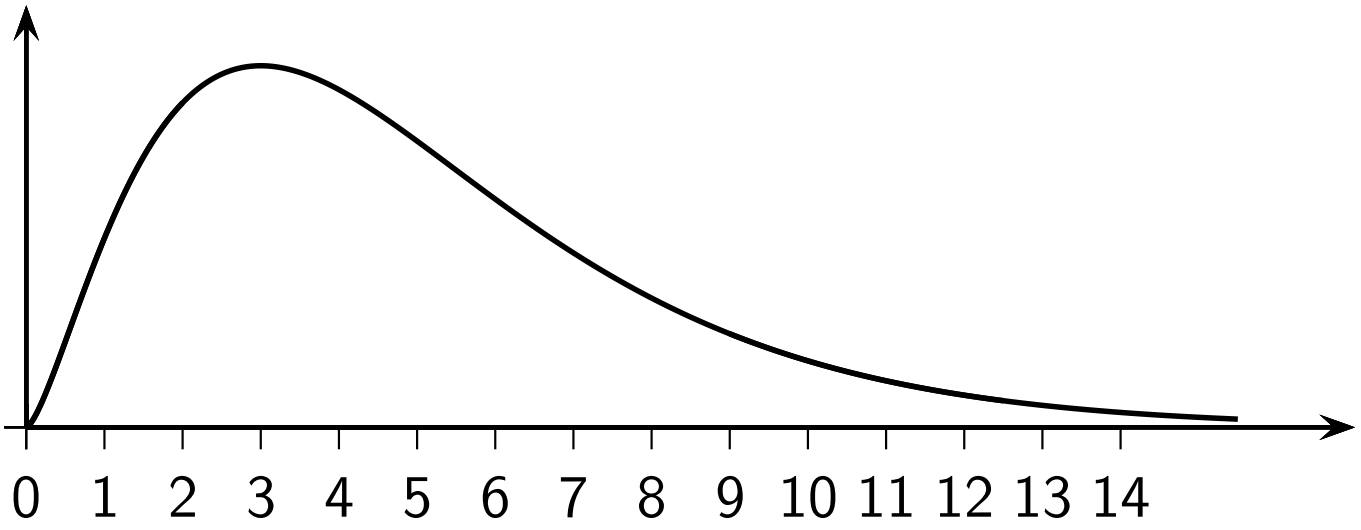
In statistics, the number of **Degrees of Freedom** is the number of values in the final calculation of a statistic that are free to vary.

What is an example of **degrees of freedom**?

Suppose I have 7 different shirts, one for each day of the week. On Monday, I have 7 shirts to choose from, and on Tuesday, I would have 6 shirts to choose from. When Saturday comes around, I still have a choice between 2 shirts.
But on Sunday, I have no choice for the shirt to wear except what is left. So I had 7-1 = 6 days of freedom in which the shirt I wear would be different from other days.

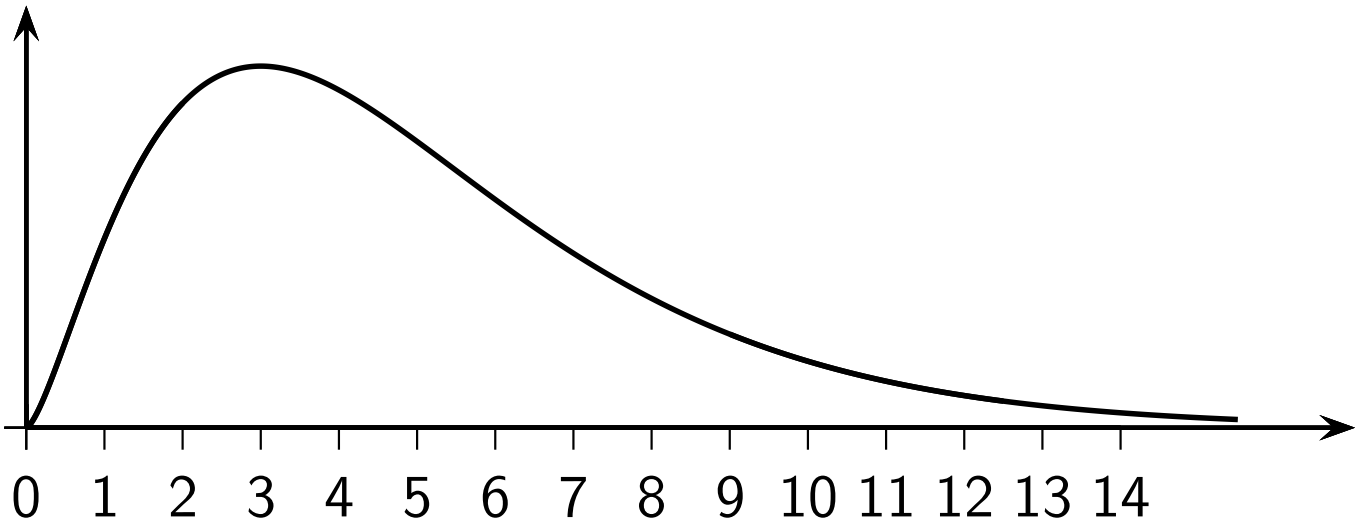
What is **Chi-Square Distribution**?

This distribution is not symmetric and the values cannot be negative. The total area under its density curve is equal to 1 and its general shape is displayed below however it depends on the number associated with degrees of freedom. As degrees of freedom increases, the shape of the graph of **Chi-Square Distribution** approaches the shape of **Normal Distribution**.



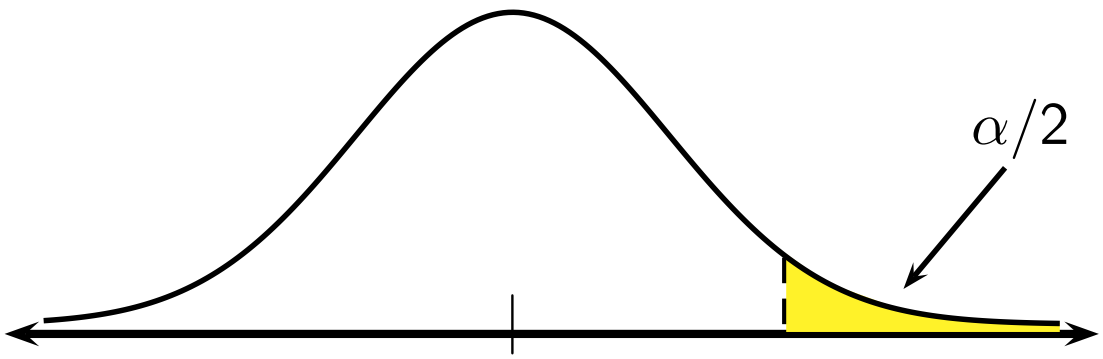
What is **F Distribution**?

This distribution is not symmetric and the values cannot be negative. The total area under its density curve is equal to 1 and its general shape is displayed below however it depends on two different degrees of freedom.



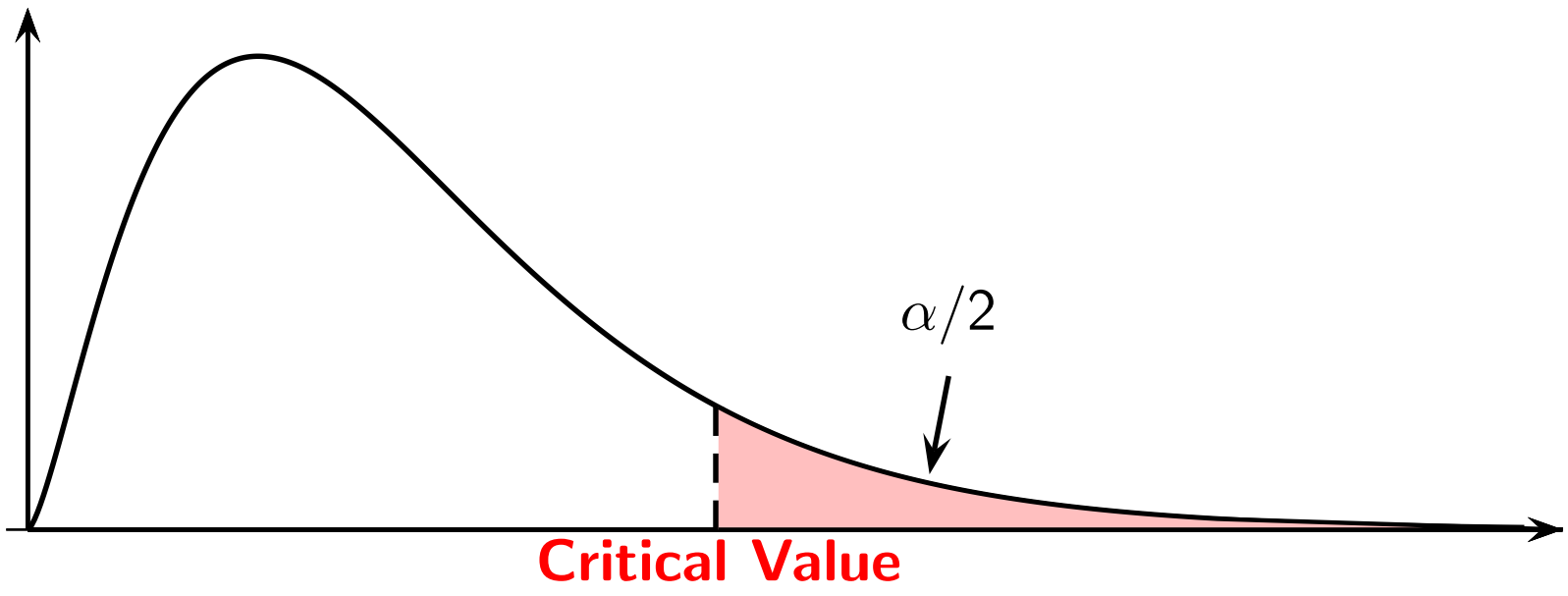
What is a **Significance Level** α?

Significance Level α is a numerical value associated with the a **right-tail** under the probability distribution curve whose area is $\frac{\alpha}{2}$.



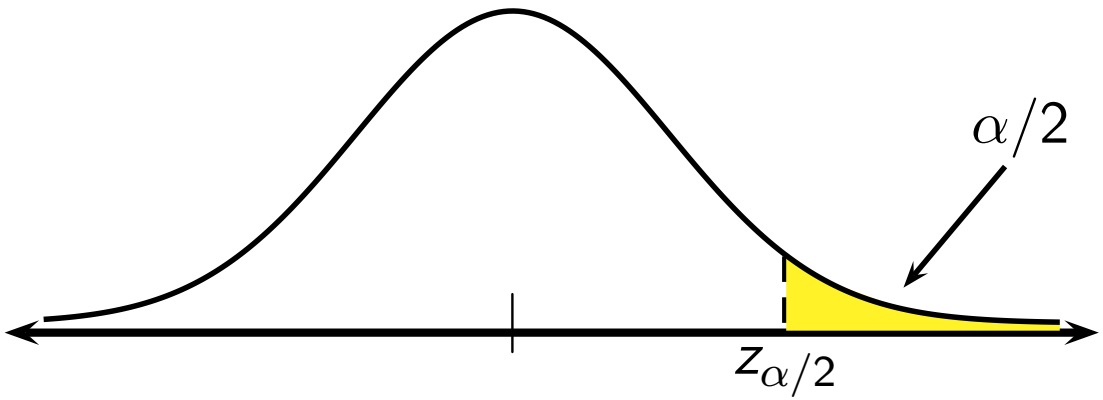
What is a **Critical Value**?

Critical Value is a numerical value that separates the right tail of the probability distribution curve with area $\alpha/2$ from the rest of the curve.



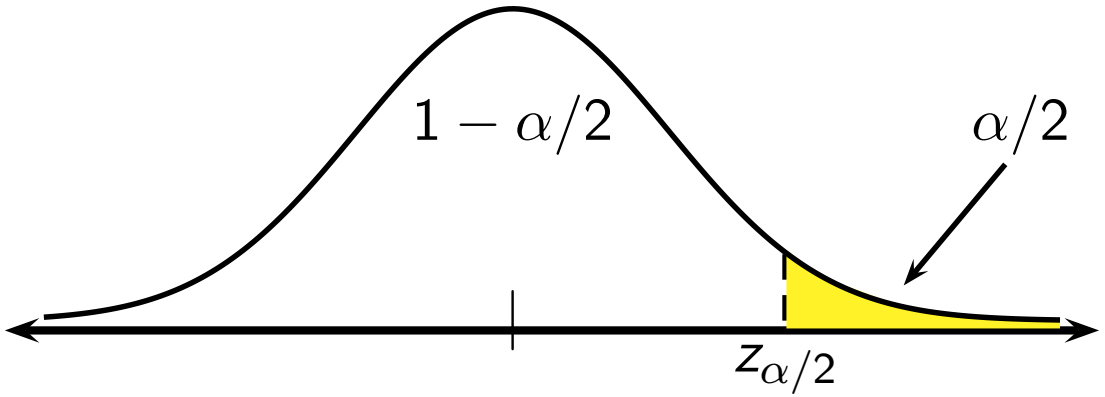
$z_{\alpha/2}$ Critical Value

- Use standard normal distribution.
- Critical Value that separates the right-tail region with area $\alpha/2$.
- Use standard normal distribution table or technology to find such critical value.



$z_{\alpha/2}$ Critical Value & TI

- Draw a standard normal distribution curve.
- Shade and label the right-tail region with area $\alpha/2$.
- Compute the left area which is $1 - \alpha/2$.
- Use **invNorm** by pressing **2nd** followed by **VARS** to proceed. For more details, please see **TI** screen shots in the next example.

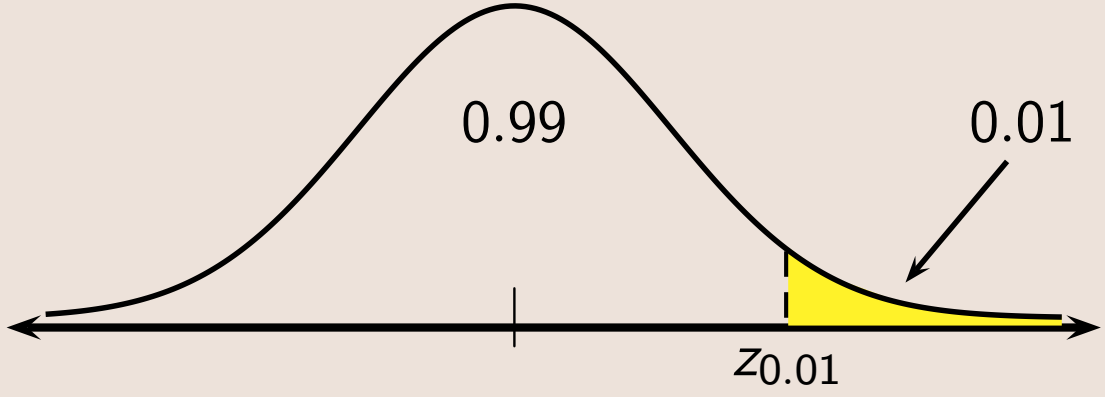


Example:

Find the critical value $z_{\alpha/2}$ for $\alpha = 0.02$.

Solution:

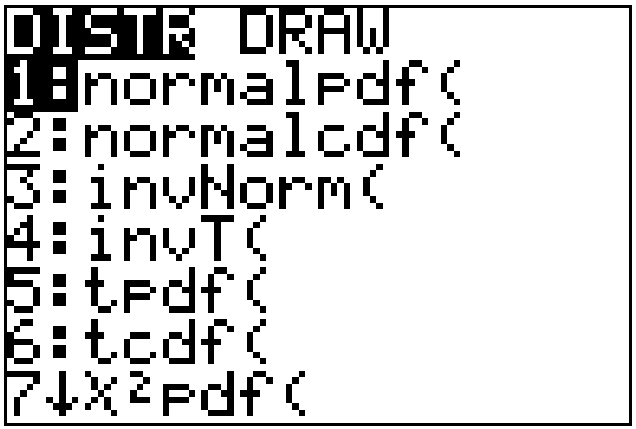
Since $\alpha = 0.02$, then the area of the right tail is $\alpha/2 = 0.01$ and the left area is $1 - \alpha/2 = 1 - 0.01 = 0.99$.



We are now ready to use the standard normal distribution table or technology to find the critical value.

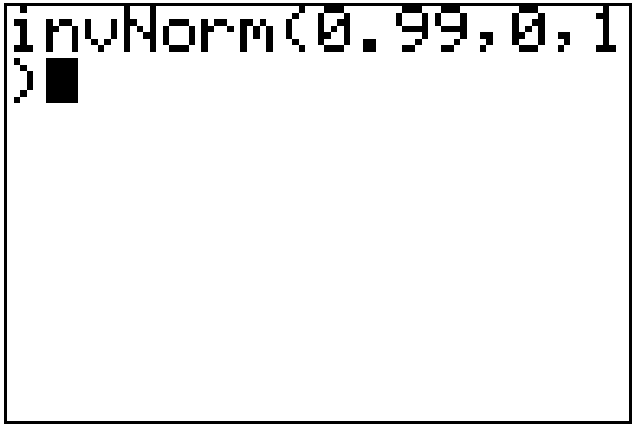
$z_{\alpha/2}$ & TI

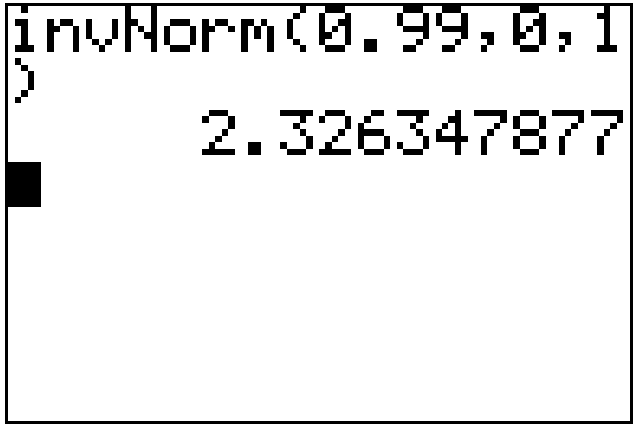
Press 2nd followed by VARS, then choose option 3 to select invNorm. If your calculator displays a menu, simply enter the required items and then press Paste followed by ENTER.



$z_{\alpha/2}$ & TI

If your calculator does not display a menu, simply enter the required items as shown below, and press ENTER.

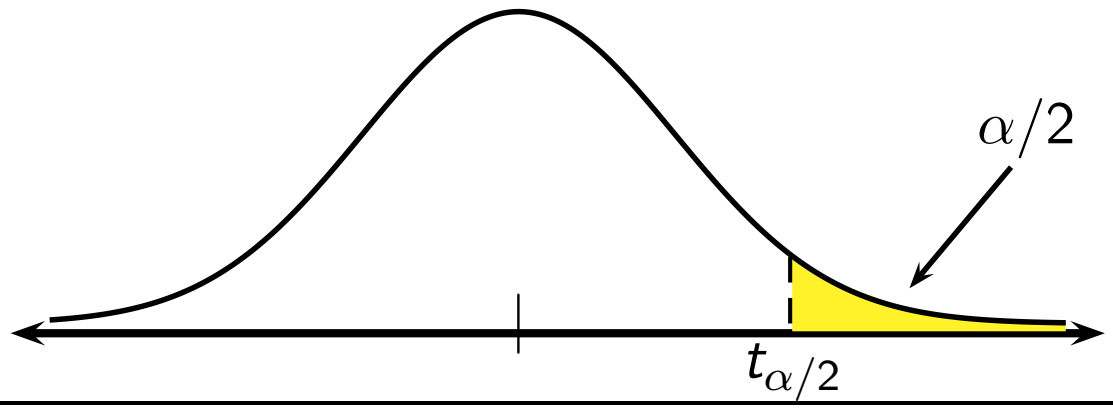




So the critical value $z_{0.01}$ is 2.326.

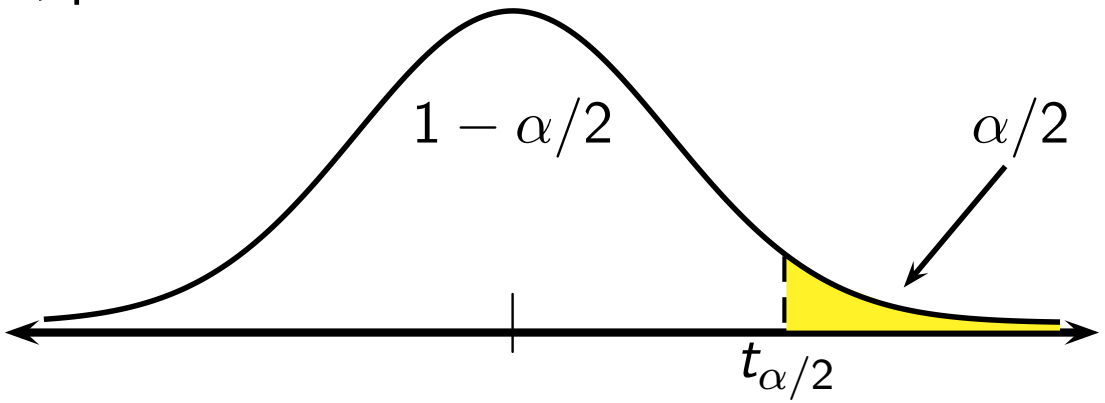
$t_{\alpha/2}$ Critical Value

- Use t-distribution.
- Critical Value that separates the right-tail region with area $\alpha/2$.
- Use t-distribution table or technology to find such critical value with a corresponding degrees of freedom.



$t_{\alpha/2}$ Critical Value & TI

- Draw a t-distribution curve.
- Shade and label the right-tail region with area $\alpha/2$.
- Compute the left area which is $1 - \alpha/2$.
- Use **invT** by pressing **2nd** followed by **VARS** to proceed. For more details, please see **TI** screen shots in the next example.

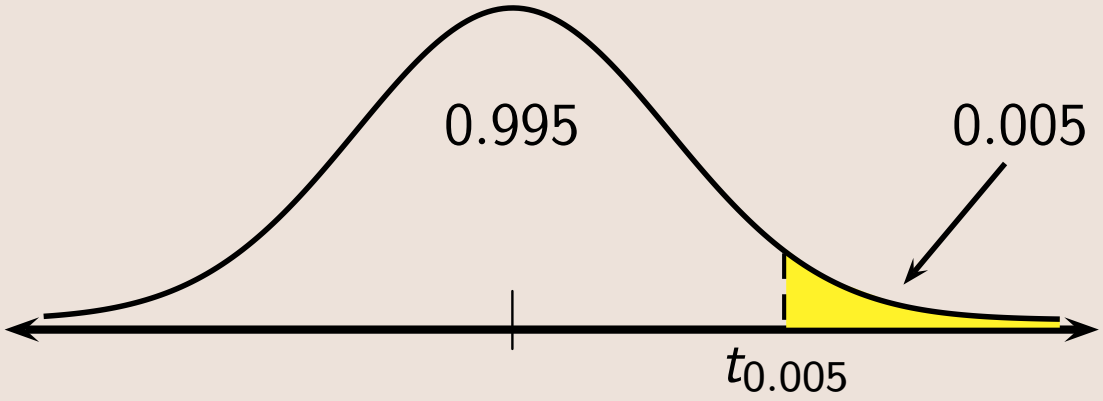


Example:

Find the critical value $t_{\alpha/2}$ for $\alpha = 0.01$ with degrees of freedom 14.

Solution:

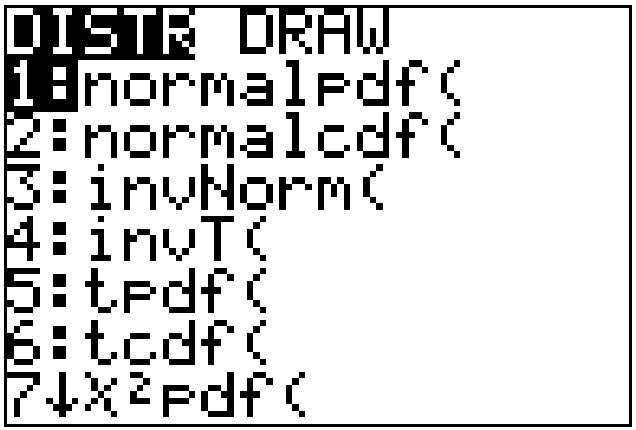
Since $\alpha = 0.01$, then the area of the right tail is $\alpha/2 = 0.005$ and the left area is $1 - \alpha/2 = 1 - 0.005 = 0.995$.



We are now ready to use the t-distribution table or technology to find the critical value.

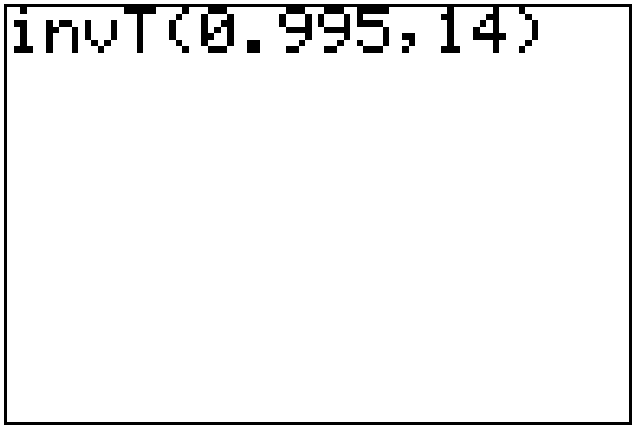
$t_{\alpha/2}$ & TI

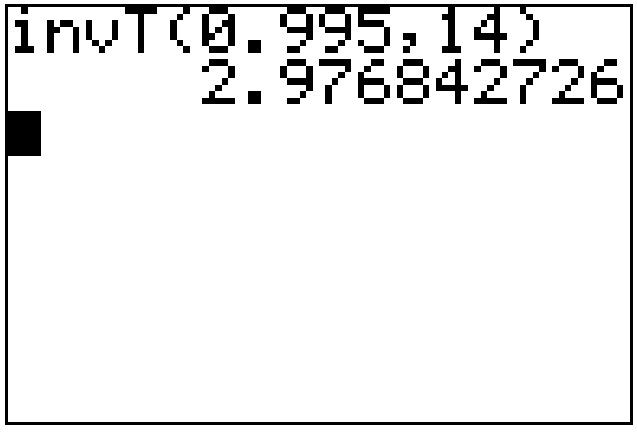
Press 2nd followed by VARS, then choose option 4 to select invT. If your calculator displays a menu, simply enter the required items and then press Paste followed by ENTER.



$t_{\alpha/2}$ & TI

If your calculator does not display a menu, simply enter the required items as shown below, and press ENTER.





So the critical value $t_{0.005}$ with $df = 14$ is 2.977.